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Spatial coherence in an open flow model

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A one-dimensional asymmetrically coupled map lattice model is studied extensively by numerical simulations. It is shown that with open boundary conditions, the one-way coupled logistic lattices can exhibit spatially uniform, but temporally chaotic states that are stable in the presence of low-level noise. It is found that the spatially uniform unstable steady state of the system can be stabilized via single point control or pinning at the boundary site. In the spatially coherent and temporally chaotic regimes, a local finite disturbance may generate a propagating localized turbulent patch, which grows in size as it is swept downstream.
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Coherent modes in spatially extended chemical, biological, and fluid systems have been the focus of considerable interest [1,2]. Theoretical studies of these phenomena include systems of coupled limit-cycle oscillators [1], the complex Ginzberg-Landau equations [1–3], and the coupled map lattices [4,5]. Recently, the coexistence of spatial coherence and temporal chaos has been observed in one-dimensional coupled map lattices with asymmetrical couplings [6]. Asymmetrically coupled map lattices (ACMLs); in particular the one-way coupled map lattices (OCMLs), have been proposed for modeling physical open-flow systems, and many interesting dynamical features have been revealed [7–11]. It has been shown that ACMLs or OCMLs may capture certain features of real open-flow systems, if appropriate boundary conditions are assumed. Many properties are found for those systems, such as noise-sustained structure, selective amplification of low-level external noise, and spatial synchronization of chaotic elements in spatially extended systems.

In this paper, we report some features of the one-dimensional asymmetrically coupled map lattice, which is defined by

$$x_{n+1}^{i}=(1-\gamma_1-\gamma_2)f(x_n^i)+\gamma_1 f(x_n^{i-1}+\gamma_2 f(x_n^{i+1}), \quad (1)$$

where x_n^i is the amplitude associated with the ith lattice point at time step n, and $i=1,2,\ldots,N$, where N is the lattice size. The local mapping function $f(x)$ is chosen to be the logisitic map, $f(x)=1-ax^2$, with the nonlinear parameter α chosen well within the chaotic regime. It is further assumed that the nearest-neighbor coupling constants obey $\gamma_1>\gamma_2\geqslant 0$. When $\gamma_2=0$, Eq. (1) reduces to the one-way coupled map lattice. For open-flow systems, the boundary conditions will strongly influence the dynamical behavior of the system. We consider the following open boundary condition:

$$x_{n+1}^{1}=(1-\gamma_2)f(x_n^1)+\gamma_2 f(x_n^2),$$
$$x_{n+1}^{N}=(1-\gamma_1)f(x_n^N)+\gamma_1 f(x_n^{N-1}).$$

For the ACML systems, it has been shown that for a wide range of the couplings γ_1 and γ_2, and with the open boundary condition, the system exhibits a stable, spatially homogeneous, and temporally chaotic state with a finite coherence length due to the numerical noise. As is shown in Ref. [6], the coherence length increases with numerical precision. Such a sensitivity to the numerical noise indicates that the spatially coherent state is unstable to dynamical local noise. We studied the ACML model through extensive numerical simulations. We found that both the asymmetry in the interactions and the open boundary condition are necessary prerequisites for the emergence of long-range-ordered chaotic states. Our numerical results show that the coupling constant γ_2, which represents the backward diffusion, plays a crucial role in the spatial amplification of small perturbations. For example, the finiteness of the coupling constant γ_2, no matter how small it is, will drastically enhance the effects of the noise, which in turn, will create a synchronized state with finite coherence length. Thus, the conclusions drawn for the case of $\gamma_2\neq 0$ are not generally applicable for the OCML systems with $\gamma_2=0$, and vice versa.

Controlling spatiotemporal chaos in spatially extended nonlinear systems with symmetrical coupling remains a challenging problem. In general, the stabilization of unstable spatiotemporal states in symmetrically coupled, spatially extended systems usually requires distributed controllers, which seems to be barely practical. Recently, the single-point control of spatiotemporal chaos has been achieved for several model systems that are convectively unstable [15–19]. Here we focus our attention on the ACML systems. The presence of backward diffusion may generally have two impacts on the dynamical behavior of the systems under consideration. On the one hand, it may certainly enhance the noise effects that will destroy the spatial coherent state and, on the other hand, it may also be used to generate certain noise-induced and/or noise-sustained dynamical behaviors. This implies that the presence of the backward diffusion may enhance the effects of the noise under some circumstances and may also suppress the amplification under other conditions. Without the backward diffusion term (as for the

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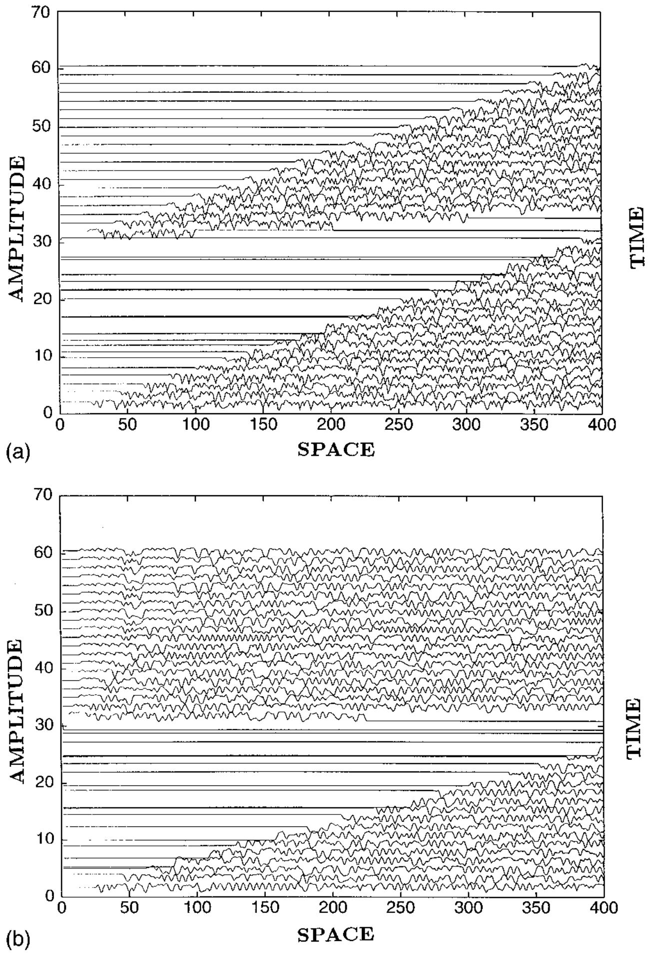

FIG. 1. Space-time evolution of the synchronization front. Iterations of Eq. (1) are plotted every 100 steps, starting from randomly chosen initial conditions. The system size is $N=400$; (a) $a=2.0$, $\gamma_1=0.75$, and $\gamma_2=0$; (b) $a=1.8$, $\gamma_1=0.5$, and $\gamma_2=0$.

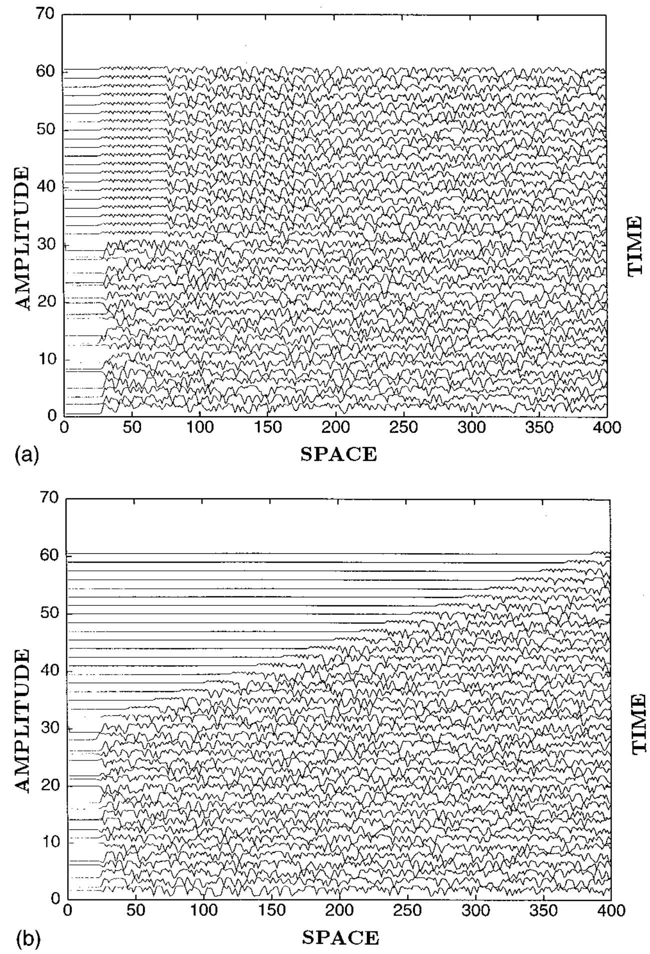

FIG. 2. (a) Synchronized state with a finite coherence length before the first site is pinned at the fixed point, and the spatial bifurcation resulting from fixing the first site at x_F, when $\gamma_2=0$; (b) the propagation of the stabilizing and synchronizing fronts when a small backward diffusion is added to the OCML system.

OCML systems), it is still possible that the noise may make the spatially uniform but temporally chaotic states unstable.

We begin with the control of spatiotemporal chaos in the ACML systems. For the OCML systems, it is well known that the unstable spatially uniform and temporally periodic states may be stabilized by fixing appropriate temporal periods at the first site of the OCML [18]. On the other hand, Kaneko found that by fixing the first site to an arbitrary constant value, many different spatiotemporal patterns can be observed. Nevertheless, no stabilization of spatiotemporally uniform states is reported. We performed numerical simulations on the OCMLs and found that the stabilization of spatiotemporally uniform states may be achieved by fixing or stabilizing the first site to the fixed point of the logistic map. In the following numerical discussion of single-point control of the spatiotemporal chaos, we first let the system operate freely, and then at the iteration step 30×100 we turn on the control and observe its influence on the dynamical behavior of the system under study. Figure 1 shows the stabilized spatiotemporal uniform state obtained by pinning the first site at the fixed point x_F, where x_F stands for the fixed point of the logistic map. That is, the existence of the spatial coherence state does not imply that such a coherent state can be controlled in a spatiotemporally uniform state. For example, at $a=1.9$ and $\gamma_1=0.5$, we have a spatially coherent state with temporal chaos that cannot be stabilized to the fixed-point state by using single-point control.

The diffusive coupling plays a very delicate role in the dynamical behavior of the coupled map lattices. When the system is in a spatially uniform state, the diffusion may tend to maintain the homogeneity, while if the system is already in a nonuniform state, it is also possible that the diffusion may enhance the effects of the noise and destroy the spatially uniform state. In what follows, we report the numerical analysis of the ACML systems. In particular, we will check how the small backward diffusion $(0\neq\gamma_2\ll1)$ influences the dynamical properties of the ACMLs. In Fig. 2, we show that a small backward diffusion can help stabilize the spatiotemporally uniform state. The system parameters are $a=2$, $N=400$, $\gamma_1=0.678$, and $\gamma_2=0.01$. In comparison with the OCMLs with the same parameter values, except that $\gamma_2=0$, it is noted that the introduction of a backward diffusion term seems to suppress the spatial amplification of the noise and makes the one-site control of spatiotemporal chaos possible. For γ_2 large, however, neither synchronized nor controlled states are observed, indicating that different magnitudes of the backward diffusions may play different roles in the development of the spatiotemporal waves. It should be emphasized that the mechanisms that lead to the propagation of synchronizing force and controlling force are quite different. This can be seen clearly in the case of $a=1.9$, $\gamma_1=0.75$, and $\gamma_2=0$, where the coherence wave can propagate throughout the entire system, but the stabilizing wave cannot. In this case, the addition of backward diffusion may destroy the spatial coherence but does not affect the single-point control of spatiotemporal chaos. Roughly speaking, if for OCML systems with $\gamma_2=0$ the unstable fixed-point state cannot be stabilized by controlling the first site, then by introducing a small backward diffusion control of spatiotemporal chaos may be achieved.

To test the stability of the spatially uniform states, we introduce a localized disturbance into the system under consideration. We have found that the small disturbances die out quickly. However, the sufficiently large, localized disturbance can propagate downstream. If the system operates at the periodic regime, then the perturbed state remains localized as it propagates, while if the system operates at the chaotic regime, the perturbed state becomes a propagating localized patch whose size grows as it is swept downstream, because the disturbance travels at a velocity that is larger than that of the synchronizing front (see Fig. 3). We analyzed the dependence of the propagation velocities of the controlling waves and the disturbance waves on the system parameters. We found that the localized disturbances propagate at a velocity (denoted by v_d) that is approximately equal to the intrinsic velocity of the coupled map lattices, i.e., $v_d=1$ site/step. While the velocities of the controlling and the synchronizing fronts, denoted by v_p and v_s, respectively, depend on all system parameters such as the nonlinearity a and the diffusive coupling constants γ_1 and γ_2, in general, the velocities v_p and v_s increase with γ_1, and decrease with a and γ_2. Thus, it is natural to expect that for γ_1 sufficiently large and a sufficiently small, one may observe the propagation of solitonlike disturbance patches.

The growing spatiotemporal spots observed in our model systems are similar to the propagating localized turbulent flashes in the flow of water down a pipe [20], where the slugs in the fluid systems are surrounded by nonturbulent fluid. In our case the spatiotemporal chaotic patches coexist with the spatially uniform, temporally chaotic surroundings in a synchronized chaotic state and coexist with the spatially uniform, temporally periodic motions in a controlled state.

In conclusion, we have shown that with open boundary condition the OCML systems may exhibit long-range spatial coherence and temporal chaos in the presence of numerical noise. We found that this spatially uniform state is stable to small perturbations. The emergence of the spatially homogeneous and temporally chaotic state is explained by a chaos synchronization mechanism. We have also revealed that for certain values of system parameters, the unstable steady state of both ACML and OCML systems may be stabilized by the application of single-point control techniques at the upstream edge of the system under consideration. No straightforward relation between the propagation of the synchronizing and controlling front is found. Our results may be regarded as a complement to the properties of ACML and OCML systems discussed in [6,9,12–14].

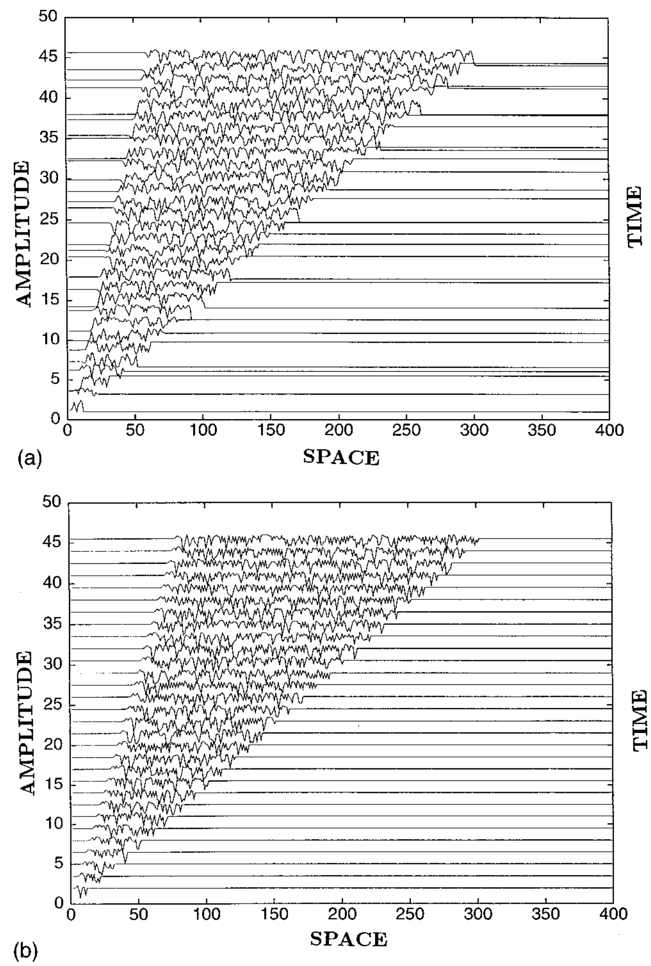

FIG. 3. Time evolution of the initial pointlike disturbance of finite amplitude at the left-hand boundary in a spatially uniform state. The system parameters are (a) $N=400$, $a=2$, $\gamma_1=0.75$, and $\gamma_2=0$; (b) $N=400$, $a=2$, $\gamma_1=0.8$, and $\gamma_2=0.01$. The amplitude of the pointlike disturbance is $\delta=0.1$, and that is switched on for a duration of five iteration steps. The states of the array are shown every ten iterations.

It is interesting to notice that in the spatial coherence state a sufficiently large localized perturbation may propagate to form a traveling spatiotemporal chaos slug or a traveling pulse or wave front, depending on whether the temporal behavior is chaotic or periodic. Since the desynchronizing force travels at a velocity greater than that of the synchronizing force, the turbulent slug grows in size as it is swept downstream.

From our numerical simulation results, we see that the backward diffusion of the perturbation [characterized by $\gamma_2\neq 0$ in Eq. (1)] may enhance the amplification of noise, resulting in a synchronized state with a finite coherence length in one case, and may suppress the effects of noise in favor of the propagation of a stabilizing front in the other. Therefore, it will be worthwhile to characterize the noise effects quantitatively in open-flow systems. Since the influence of the diffusive coupling on the dynamical behavior of spatially extended systems with convective instabilities is rather intricate, it is not clear as to whether or not the one-dimensional results could be generalized to ACMLs in higher dimensions.

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